Implicit Differentiation

explicit - solved for the dependent variable

$y = x^2 + x + 2$

implicit - not solved for dependent.

x and y are on the same side of the equation. $x^2 + 2y^2 - 3 = 0$

$$x^2 - 2y^3 + 4y = 2$$

- You are still differentiating with respect to x. $\frac{dy}{dx}$
- Terms that involve x alone are differentiated as usual.
- Terms that that involve y must be differentiated using the Chain Rule

Remember that a function is not differentiable at points with vertical tangent lines and at points where the function is not continuous.

$\frac{d}{dx}[x^3] = 3x^2$

$\frac{d}{dx}[y^3] = 3y^2\frac{dy}{dx}$

Chain Rule

outside der. · inside function

$\frac{d}{dx}y = x^2 + 3x + y$

$y' = 2x + 3$

$$\frac{d}{dx}[x+3y]$$

$$1 + 3y^0 \frac{dy}{dx}$$

$$1 + 3\frac{dy}{dx}$$

$$\frac{d}{dx}[xy^2]$$

$$x\left[2y\frac{dy}{dx}\right] + y^2(1)$$

$$2xy\frac{dy}{dx} + y^2$$

$$\frac{d}{dx}y^3 + y^2 - 5y - x^2 = \frac{d}{dx}4$$

$$3y^2\frac{dy}{dx} + 2y\frac{dy}{dx} - 5\frac{dy}{dx} - 2x = 0$$

$$3y^2\frac{dy}{dx} + 2y\frac{dy}{dx} - 5\frac{dy}{dx} = 2x$$

$$\frac{\frac{dy}{dx}(3y^2+2y-5)}{3y^2+2y-5} = \frac{2x}{3y^2+2y-5}$$

$$\boxed{\frac{dy}{dx} = \frac{2x}{3y^2+2y-5}}$$

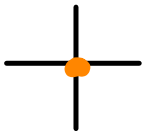
1. Differentiate both sides *with respect to x*

2. Collect all dy/dx terms on the left and other terms on the right.

3. Factor out dy/dx

4. Solve for dy/dx

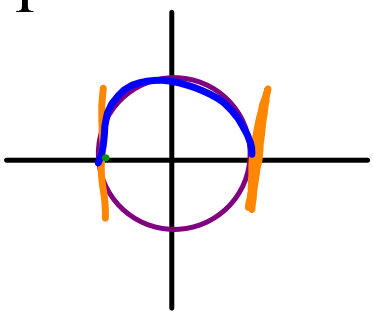
$x^2 + y^2 = 0$



$x^2 + y^2 = 1$

$\sqrt{y^2} = \sqrt{1-x^2}$

$y = \pm\sqrt{1-x^2}$



$(-1, 0)$

$\frac{d}{dx} x^2 + y^2 = 1$

$2x + 2y\frac{dy}{dx} = 0$

$2y\frac{dy}{dx} = -2x$

$\frac{dy}{dx} = \frac{-2x}{2y}$

$\frac{dy}{dx} = -\frac{x}{y}$

$$x+y^2=1$$

$$(x-y)^2 = x+y-1$$

Determine the slope of the tangent line to the graph of

$\frac{d}{dx}\ x^2+4y^2=4\ \frac{d}{dx}$ at $(\sqrt{2},\frac{-1}{\sqrt{2}})$

$2x + 8y\frac{dy}{dx} = 0$

$8y\frac{dy}{dx} = -2x$

$\frac{dy}{dx} = \frac{-2x}{8y}$

$\frac{dy}{dx} = \frac{-x}{4y}$

$m = \frac{-x}{4y} = \frac{-\sqrt{2}}{4\left(\frac{-1}{\sqrt{2}}\right)} = \frac{1}{2}$

Second derivative

$x^2+y^2=25$